

12-1/12-2 Randomness and Probability

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M11/5/MATHL/HP1/ENG/TZ1/XX

Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

12. [Maximum mark: 19]

Consider the function $f(x) = \frac{\ln x}{x}$, $0 < x < e^2$.

- (a) (i) Solve the equation $f'(x) = 0$.
- (ii) Hence show the graph of f has a local maximum.
- (iii) Write down the range of the function f . [5 marks]
- (b) Show that there is a point of inflexion on the graph and determine its coordinates. [5 marks]
- (c) Sketch the graph of $y = f(x)$, indicating clearly the asymptote, x -intercept and the local maximum. [3 marks]
- (d) Now consider the functions $g(x) = \frac{\ln|x|}{x}$ and $h(x) = \frac{\ln|x|}{|x|}$, where $0 < |x| < e^2$.
- (i) Sketch the graph of $y = g(x)$.
- (ii) Write down the range of g .
- (iii) Find the values of x such that $h(x) > g(x)$. [6 marks]

The Sample Space S of a random experiment is the set of all possible outcomes.

Flip a coin twice. Sample space:

$\{HH, TH, HT, TT\}$

Roll 2 dice and find the sum. Sample space:

$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

A Simple Event is the outcome we observe in a single repetition (trial) of the experiment.

An Event is an outcome or a set of outcomes of a random experiment.

Ex1. Roll two dice and find the sum.

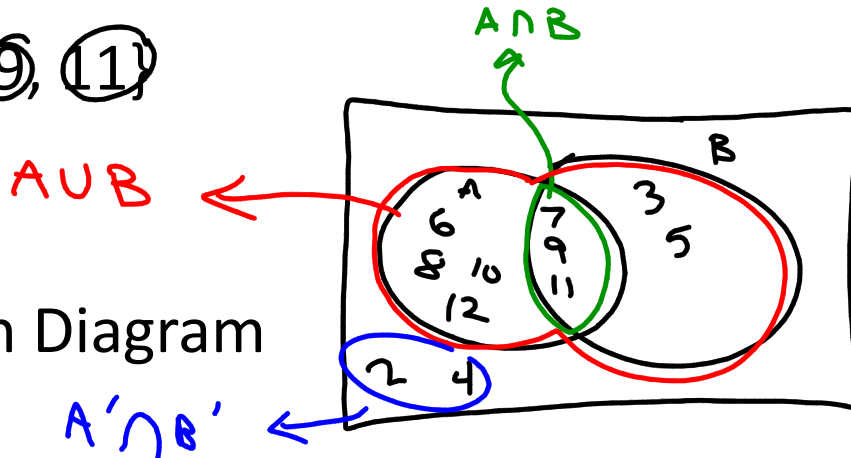
Let event A = the sum is 6 or greater

Let event B = odd sum

$$A = \{6, 7, 8, 9, 10, 11, 12\}$$

$$B = \{3, 5, 7, 9, 11\}$$

Draw a Venn Diagram



Find:

1.) Compliment of A (A') $A' = \{2, 3, 4, 5\}$

2.) A Intersect B ($A \cap B$) $\{7, 9, 11\}$

3.) A union B ($A \cup B$) $\{3, 5, 6, 7, 8, 9, 10, 12\}$

4.) Compliment of B (B') $\{2, 4, 6, 8, 10, 12\}$

5.) $A' \cap B = \{3, 5\}$

7.) $A' \cap B' = \{2, 4\}$

9.) $A \cup B' = \{2, 4, 6, 7, 8, 9, 10, 11, 12\}$

11.) $(A \cup B)' = \{2, 4\}$

13.) $(A \cap B)' = \{2, 3, 4, 5, 6, 8, 10, 12\}$

6.) $A \cap B' = \{6, 8, 10, 12\}$

8.) $A' \cup B = \{2, 3, 4, 5, 7, 9, 11\}$

10.) $A' \cup B' = \{2, 3, 4, 5, 6, 8, 10, 12\}$

12.) $(A' \cup B')' = \{7, 9, 11\}$

14.) $(A' \cap B')' = \{3, 5, 6, 7, 8, 9, 10, 12\}$

DeMorgan's Law

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Two events that have no outcomes in common and hence can never occur together are called disjoint events or mutually exclusive events.

Ex2. You roll two dice and find the sum. Find the probability of each of the possible outcomes.

		1 st roll					
		1	2	3	4	5	6
2 nd roll	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

36
equally likely
possibilities

-	2: (1,1)	$P(2) = \frac{1}{36}$
-	3: (1,2), (2,1)	$P(3) = \frac{2}{36}$
-	4: (1,3), (3,1), (2,2)	$P(4) = \frac{3}{36}$
-	5: (1,4), (4,1), (3,2), (2,3)	$P(5) = \frac{4}{36}$
-	6: (1,5), (5,1), (4,2), (2,4), (3,3)	$P(6) = \frac{5}{36}$
-	7: (6,1), (1,6), (5,2), (2,5), (3,4), (4,3)	$P(7) = \frac{6}{36}$
-	8: (6,2), (2,6), (5,3), (3,5), (4,4)	$P(8) = \frac{5}{36}$
-	9: (6,3), (3,6), (5,4), (4,5)	$P(9) = \frac{4}{36}$
-	10: (6,4), (4,6), (5,5)	$P(10) = \frac{3}{36}$
-	11: (6,5), (5,6)	$P(11) = \frac{2}{36}$
-	12: (6,6)	$P(12) = \frac{1}{36}$

Find the probability:

$$P(\text{sum} > 10) = \frac{3}{36} = \frac{1}{12}$$

$$P(\text{sum is even}) = \frac{18}{36} = \frac{1}{2}$$

$$P(\text{sum is prime}) = \frac{15}{36} = \frac{5}{12}$$

A = the sum is even B = the sum is prime

Find the probability:

$$P(A \cap B) = \frac{1}{36}$$

{2}

$$P(A \cap B') = \frac{17}{36}$$

{4, 6, 8, 10, 12}

$$P(A' \cap B) = \frac{14}{36} = \frac{7}{18}$$

{3, 5, 7, 11}

$$P(A' \cap B') = \frac{1}{9}$$

{9}

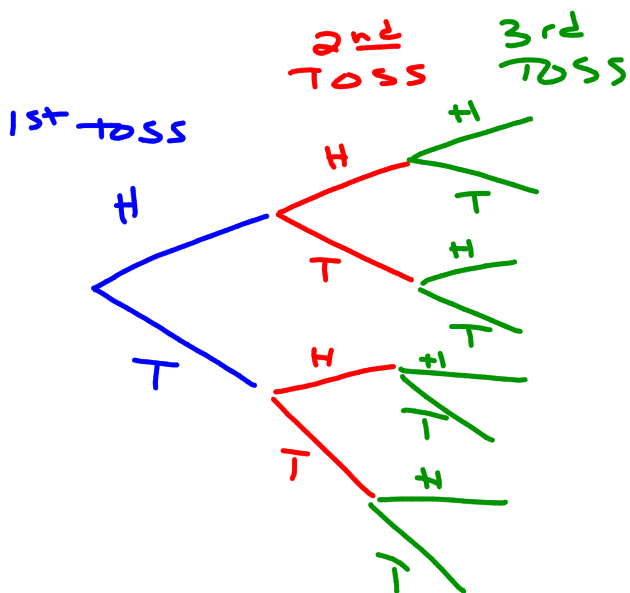
$$P(A \cup B) = 1 - \frac{1}{9} = \frac{8}{9}$$

$$P(A \cup B') = \frac{22}{36}$$

$$P(A' \cup B) = \frac{19}{36}$$

$$P(A' \cup B') = \frac{35}{36}$$

Ex. A coin is tossed 3 times. Draw a tree diagram.



8 outcomes

$$2^3$$

Find:

- The sample space
- The probability of getting 3 tails
- The probability of getting a least 2 heads

HW pg 524 # 2, 3, 5-7, 10, 11, 14